Werner Depauli-Schimanovich (FN1) footnote1: My old address is still valid: c/o Institut für Statistik und Informatik der Universität Wien/Austria.

Class-Theory and Philosophical Set-Theory

Classes are predicate-extensions, i.e. entities of all individuals or objects x which fulfil a certain property, say E(x). You can either enumerate them: {|x-sub(01), x-sub(02), x-sub(03), . . . |} [recognize: I signify constants always with a sub-index beginning with zero 0, or with a combination of letters.] And classes are expressed by capital letters: A, B, C, . . . X, Y, Z, also with indices, as variables. But note: the elements of classes can in general only be sets: Set(X):= (E)Y: X in Y, where "(E)" means the Existenz-Quantor (existence quantifier) and "in" symbolizes the elementhood "(.)" of a class. As usual convention we have: sets are denoted by small Latin letters a, b, c, . . . x, y, z, also with indices. The second possibility to express a class is defining [or fixing] it by extension [of some property E(x):] {| x: E(x) |}, where E can contain also other free variables [than x]. IN MY NOTATION the class operator is always symbolized by a vertical bar together with the set parentheses. As the famous Russell-paradox does us show: Not every class can extensionally be a set, but if we consider the set-constituting predicate intentionally (i.e. if we take its intended meaning), also the proper-classes can be converted into sets. That will be explained in a separate paper "On Frege's correct way out."

Now remains the question: how can I know that a given class is a set? Usually I have either a more or less arbitrary enumeration of axioms. E.g. in the well known Zermelo-Fraenkel+Skolem set theory ZFS [which should correctly be named ZSF, because Skolem added the axiom of replacement [[today more commonly called image-set axiom]] as first logician to the Zermelo-axioms.] Concerning ZFS please consult the literature! But ZFS is good for logicians, and fortunately uninteresting for mathematicians. There exist about 100 axiomatic systems of set theory or more! It is very hard to decide which one should be the most (philosophically) relevant one. The most famous of them is (in my opinion) "New Foundation" NF of Willard van Orman Quine. Unfortunately NF has not been accepted by the mathematicians. And for philosophers it was too complicated. But with proceeding of time the old imperative and conjecture of Hausdorff and Fraenkel (i.e.: contradictions in set theory arise only through the construction of sets which are extensional equal to the universe) and this is the universal class V/ or eventually the set v/ [the slash shall us remember that these 2 entities are constants]. (Ronald Björn Jensen did show 1969 that ZFS is deductionally equivalent with NFU = NF plus “Urelements”.) Please confer the important book “set theories with a universal set” of my friend and Cambridge professor Thomas Forster. My favourite mathematical system is NFUM = “NF with urelements and a measurable ordinal class O)n [the closing parenthesis remembers us only to the fact that this is a constant] designed by Randall Holmes under cooperation of Robert Solovay. A group of young Russians at the British Bristol university could demonstrate with NFUM that certain statements of Ramsey theory are not decidable, what is not possible with ZFS. They presented this result in a talk given at a small conference organized by Matthias Baaz, the general secretary of the international Kurt Gödel Society KGS [founded by my small personality, I am known because of the film I produced (together with the artist Peter Weibel from the ZKM in Karlsruhe, Germany) and the booklet (I published together with the American science writer John Casti) ”Gödel: A Life of Logic”]. Both the film and a booklet (together with Peter Weibel) in German (but with a lot of documentary photo-material) have the title “Kurt Gödel: Ein mathematischer Mythos”. From the booklet “Gödel: A Life of Logic” I sold already more than 200.000 issues only in the USA. This book is written for the general Gödel Society KGS reader, only the last chapter is for mathematicians. Before we end the mathematical introduction, I want to call you in mind: It is important to keep always in memory, that from the philosophical point of view: a pure collection of objects has not automatically existence, even if it is consistent! (But I confess, that sometimes I take the freedom to use this “Hilbert rule” (that consistency implies existence), and do establish existence because of consistency. It is still important to explore the broad field of consistency and coexistence [category theory does not contribute to this problem].

Now let’s go to philosophy: mathematicians work more or less until today with naïve set theory (German: Mengenlehre), abbreviated NST (or: NM). The intuition of mathematicians (can be fallable) but is usually so strong, that no contradictions arised since 100 years. We can consider this fact as a de facto proof of naive consistency of set theory NST. This is also the reason why Bourbaki always ignored ZFS and the fear of inconsistencies! [Confer also Ludwig Wittgenstein’s posthum published work edited by Mac

Guinness et al.] But when NST is (experimentally) consistent, it must also be possible to describe it formally. I tried to do this and design some systems, not obligatory axiomatic, the reader can find in my books EUROPOLIS6, partially in Europolis5 or 4, and in my blog at You Tube (I hope soon.) (FN2) Footnote2: Unfortunately you can find my books only in a library, or you have to order them from US-antiquariats or at Amazon, because in Europe all my books are sold out.

If we (concerning my conviction) consider it exactly, then there is principally NO “strict axiomatic system” which enumerates the logical proper-classes (i.e. those already produced by the class theoretical frame CTF of only the class-operator and the axiom of extensionality together with PIF-logic exactly defined later), because that would contradict the famous theorem of Gödel. Such a system would make set theory decidable, what’s impossible, because it contains arithmetics and all systems containing it are uncompleteable. See my article “A Brief History of Future Set Theory” in Europolis6, pages 589 until 595.

Buy or borough the book Europolis6 “Informatik für Spiele und Verkehr. Extension der Mengenlehre“. May be that I can transfer this text into Facebook, because the Kurt KGS cannot publish it in its yearbook, cause the Ministery for Science does not pay any subvention. (They are bankrupt.) My blog in Facebook is “ZFK = ZF``+ Komplement”. In Europolis6 are the articles quoted in the following. Now I will continue and explain you first the main ideas and methods of philosophical set theory. The idea is simple: How can we convert every class, which is NO proper-class (by logical means alone) into a set? That sounds simple, but is very sophisticated, also from a philosophical point of view!

Therefore I divided the logical paradoxes [i. e. not the semantical ones] into 3 disjoint groups: (1) the first ones are the pathologies, which are mainly based on proper-classes [please consult Europolis6, page 605 to 623.] (2) the second are the proper-antinomies, which disappear when you cancel the concerning axiom(s), (3) the counter-intuitive but not contra-dictionary paradoxes. [please consult Europolis6, page 639 or 647until 659: Naïve Axiomatic Mengenlehre for Experiments.] The problem is only group (1) but I hope that I found the solution to characterize [if not formalize] it.

Please see Europolis6, page 624 to 628: Naïve Axiomatic Class Theory. To say it simple: that are all classes which have no circle in its element relation and no infinite descending element sequence. But this notion is not easy to describe formally. The second group (2) is easy: that are the contradictions, which arise because of a certain axiom of set theory. E.g. Cantor's contradiction of the universal set, which is only following from the axiom of replacement (resp. from the axiom of separation). The third group (3) are the abnormal sets. I construct a system for experimental set theory (please read the article "Naïve Axiomatic Mengenlehre NAM for Experiments" in Europolis6, page 647 until 659.), which has a prime predicate "normal", and always a contradiction arises this set becomes "abnormal."

I also tried to give some philosophically plausible statements (or call them axioms) for establishing "normal" sets. E. g. for each predicate/property P either P or non-P produces a set. Or: With every small set, also its complement is a set. I prefer to use "slim" [i.e. smaller mighty than its complement] for small, but other properties like well-founded, Cantorian, etc. are also allowed. The reader can see that here is opening a large interesting field for philosophy. Unfortunately it is for mathematicians, but it is good to know some mathematics or better mathematical logic, especially axiomatic set theory! We shall explore consistencical co-existence.

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Adios, good bye, and good luck for your investigations. Because now follow some personal words to understand better my personality and why I involved myself into such crazy-making questions. My real job now is composer. Please look at Youtube: Put "depauli-schimanovich into the search-slot, then you will get a part of my compositions especially the hymns, not always for distribution [e. g. the strophes of the "Pan-Euro-Community" had been sent to a friend of my younger daughter in San Francisco and he made the Pan-Euro-March. That is excellent but completely unknown. The time is not ripe for that. But in 50 years it will be. I will be dead already. This P.S. was necessary to convince the reader that I am a Renascence man and “see-er”. It is very difficult to judge my ideas correctly, but 80% of my predictions became true.

LITERATURE: See Europols6, pages 691 until 693.

The list of quoted articles also from Europolis6 are:

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A nice contribution to the complex of this problems is the preface of ”The Philosophy of Bertrand Russell”, a book that is certainly difficult to borough somewhere, but I am fortunately one of the minimal number of owners of this book.