Speech for the collegium logicum vindobonensis of the kgs from January 2020

Terminology: A, B, C, . . . X, Y, Z, . . . Variables for classes (of course also with indices). “in” is the element relation for classes. Small Latin letters are by general convention variables or constants or function-names, also 2 or 3 letters together, for sets.

Heterological (= the contrary of autological) == not-self-applicable, abbreviated NSA. E. g.: if E is any wff (= well-formed formula), then NSA(E) := not-E({| x: E(x) |}). E. g. that’s when E is x =/= x, or: Set(X) when X is a proper-class, etc.

Set-brackets “{“ and “}” without any other symbol signify the normal sets of Naïve Mengenlehre (I use the German word sometimes, because set theory was a German invention, and the English native speakers should be remembered to this fact!) But be careful: the set-brackets together with a vertical bar symbolize always classes, other authors like e. g. Ebbinghaus use corner-brackets ”[“ and “]”, but they are in math usually used for equivalents-classes (i. e. classes of objects which are reflexive, symmetric and transitive to each other). Therefore we prefer to use set-brackets together with vertical bar that reminds us also that sets are classes. And set-brackets together with a slash symbolize classes of classes (i.e. 2nd order classes), and to denote a special system you can add 2 or 3 capital Latin letters (e. g. “{ZF” and “ZF}” or: “{NBG” and “NBG}”).

Now let E be again some expression (wff), then we want here to try to give the reader an attempt for a formal characterization of the terminus “intension”. Intension(E) is the intended meaning of E. This is of course dependent from the context. (Some examples will make it clear in the following.)

The intension of a class is often specified by the Church schema CS, i. e. in general the formula:

(A) wff E: (A) x: x in {|y: E(y) |} <==> E(x). This is usually only valid for classes. (remember: sets are classes.) We define with Willard van Orman Quine: Set(X):= (E)Y: X in Y, and write little Latin letters for them. (This definition is the only case of quantification of classes in NBG, but there exist other set theories, like e. g. the Morse-Kelly class theory where also other quantifications are allowed.) The well-known Russell-(Proper)-class Ru:= {|x: x not-in x|}, is an excellent example to explain the meaning of the notion intension, because already in history there had been some experiments to formalize this intended meaning, starting with Gottlob Frege himself, who failed with his attempt to define the Russell-class as a set: ru:= {F x: x not-in x F} with y in ru <==> y not-in y and y =/= ru. [the F together with the set- brackets indicate that here is not the usual CS applied, but Frege’s failed attempt (way out) to overcome the contradiction.] There followed papers by Quine, Geach and later by Hintikka with title “On Frege’s true way out”, where they defined a “set” {TW x: x not-in x TW} but they had also a wrong intension. So I hope I will have more luck, when I define the “correct way out”: ruCW:= {CW x: x not-in x CW}, where my intension is: y in ruCW <==> y not-in y &y =/= ruCW & y not-in *y & y not-Miri, where y in *z := Uw & w := y´`i & (i in Nat|n [:= 0, 1, 2, 3, . . . n-1]; y Miri := y has a descending element-sequence).With this example we can feel what “intension” means, but also how difficult it is to find the correct formalization.

I propose to differ between mathematical set theory and the philosophical one, even if we may need more knowledge of mathematics to operate the philosophical set theory, than we need in the mathematical one.

My definition of philosophical set theory := the attempt to convert as much classes as possible into sets (by finding its correct intension). E. g. all classes of NBG or NF (Quine's New Foundation, and especially NFUM (by Randall Holmes under cooperation of Robert Soloway) = NF with Urelements and a Measurable Ordinal-class On. This is equivalent to convert all proper-classes into sets. The correct intension of a wff to form a set is not exactly definable. Since Gödel it is also not provable to be consistent. But that is nowadays not so important; we need only statistical consistency, or as I defined it in the Proceedings of the 3rd Wittgenstein-Symposium as nearly-consistency, i. e. that the relative frequency of the "prime-contras" (i. e. the "first-contradictions") converge to zero 0 compared with all wffs of a suitable production system PS. [A similar situation occurs today with measurability. In the age of computation we need only statistical results. And after Gregory Chaitin's randomness of arithmetics this should be clear for everybody. Also in Physics, Quantum theory teaches us this!] Here is certainly a paradigms shift going on. Kurt Gödel proposed that math will develop itself in future more like physics (i.e. become more and more an applied science).

So from my point of view, my thesis that:

"Philosophical Set Theory == Hereditary-Heterological Class Theory "

Is really revolutionary! And I gave also some simple principles to construct a hereditary-heterological class theory. These principles are published in my book Europolis6 "Informatik für Spiele und Verkehr. Extension der Mengenlehre." (I made also some copies out of the book and corrected it with OCR-software. But if you can, please order the book from some USA-antiquariate [because in Europe all my books are sold out.] Otherwise you have to borough it from a library!

Some of my philosophical principles PP are:

1st PP: (A) wff E: eather {| x: E(x)|} or {|x: Not-E(x) |} is a set. One of some exceptions: The property to be an ordinal number On(x). [Because in NBG It can be shown that also the complement of the ordinal-class is a proper-class.]

2nd PP: (A) X: Small(X) ==> Set(X) & Set(ko(X), where Ko(X):=the complement of X, and ko(X) when the complement certainly form a set. And where Small can be:

2a: Slim, where Slim(X):= card (X) < card(Ko(X)),

Or: 2b: Hwf(X):= hereditary well-founded.

Or: 2c: Cantorian(X),

Or: etc.

3rd PP: The hereditary-not-patho predicate extensions [HNP-PE] are sets. This is too difficult to explain here in a talk. Therefore I have to ask the honourful reader to consult Europolis6, pages 617ff.

Furthermore we define: Slim := {|x: Slim(x)|} and Mighty :={| x: Mighty(X)|}, and Medium := {| x: Medium(x) |}, where Medium (X) :<==> card(X) = card(Ko(X)). We see: Ko(Medium) == Slim u Mighty := KMedium. The NACT*-conjecture is the following:

Main-PP: “The co-not-equipollents or the co-proper-classes or the not-co-selfcontainers are sets.” Probably “Exactly”, if they are Medium, i. e. not already Slim or Mighty.

The reason for this is the following: The classes in Slim and Mighty are clear: They can be sets without danger of inconsistencies. But about the classes in Medium we know nearly nothing. Only one fact is important: Consider MediumC := {| x: x in Medium & ko(x) in Medium |}. I want to establish the following conjecture: Patho = = MediumNC := Medium minus MediumC. If this conjecture can be proven, it would be the first set-theoretical characterization of Patho.

Unfortunately Cantor’s definition of a set was completely wrong. It was the definition of a class; but classes are not automatically elements, and this lead to the contradictions. Even a genius such as Cantor can make once a mistake.

Therefore we have to go back to the start-position: i. e. the question: “What is a set?” (Dos pasos adelante, un paso adetras!)

I hope that my conjecture will give an answer to this fundamental question. Thanx a lot (= a set)!